

# Haldane Sashes in Quantum Hall Spectra

A.H. MacDonald

Department of Physics, The University of Texas at Austin, Austin, Texas 78712, USA

We show that the low-temperature sash features in the lowest Landau-level (LLL) tunneling density-of-states (TDOS) recently discovered by Dial and Ashoori are intimately related to the discrete Haldane-pseudopotential interaction energy scales that govern fractional quantum Hall physics. Our analysis is based on expressions for the tunneling density-of-states which become exact at filling factors close to  $\nu = 0$  and  $\nu = 1$ , where the sash structure is most prominent. We comment on other aspects of LLL correlation physics that can be revealed by accurate temperature-dependent tunneling data.

PACS numbers:

*Introduction*—Many-electron correlation physics within a partially filled Landau level has been an enduring source of new physics for several decades. In recent years the even-denominator incompressible states[1] that occur when the  $n = 1$  Landau level is partially filled, and the smectic states[2] that occur for  $n > 1$  have received particular attention. Correlation physics in the quantum Hall regime has most frequently been probed experimentally by studying transport properties of very high quality GaAs/(Al,Ga)As two-dimensional electron gases (2DEGs) and identifying the filling factors at which the quantum Hall effect occurs. The quantum Hall effect is a transport anomaly characterized by plateaus in the Hall conductance and vanishing longitudinal resistance over a finite range of magnetic field strength or electron density. Its occurrence signals[3] a jump in the chemical potential at a density which depends on magnetic field, and localization of the gapped charged excitations of the corresponding incompressible state.

Many of the correlated electron states that occur in quantum Hall systems do not have parallels elsewhere in physics and represent qualitatively new classes of electron behavior. Some, including notably the  $\nu = 1/2$  composite-fermion liquid state[4] state, are not characterized by charge gaps. Others have gaps, but also have poorly understood properties unrelated to their low-energy charged excitations. In both cases transport is an unsatisfying probe of the properties of interest. Unfortunately experiments that are complementary to transport have often not been readily available, mostly because of experimental challenges associated with the buried 2DEG location. The present article is motivated by the  $n = 0$  tunneling spectra observations recently reported by Dial and Ashoori[5] which have revealed, in addition to the Coulomb gap behavior already evident in earlier data[6, 7], a pattern of sharp structure at high energies that is most pronounced for filling factors near  $\nu = 0$  and  $\nu = 1$ . The authors refer to these unanticipated structures as *sashes*. In this Letter we show that they occur because correlation physics in the quantum Hall regime is governed by a small set of discrete energy scales ( $V_m$ ) known as Haldane pseudopotentials[3, 8].

*TDOS at small Landau level filling factor*—We start by presenting some exact results for the small  $\nu$ ,  $T = 0$ ,

limit of the LLL TDOS ( $A(\epsilon)$ ) of a disorder-free 2DEG. The TDOS is the sum of electron-addition and electron-removal contributions given at  $T = 0$  by[9, 10]

$$A_+(\epsilon) = \sum_n |\langle \Psi_n(N+1) | c_m^\dagger | \Psi_0(N) \rangle|^2 \delta(\epsilon - E_{n,0}), \quad (1)$$

and

$$A_-(\epsilon) = \sum_n |\langle \Psi_n(N-1) | c_m | \Psi_0(N) \rangle|^2 \delta(\epsilon + E_{n,0}). \quad (2)$$

Here  $|\Psi_n(N \pm 1)\rangle$  are exact eigenstates of the  $N \pm 1$  particle system,  $|\Psi_0(N)\rangle$  is the ground state of the  $N$ -particle system, and  $E_{n,0} = E_n(N \pm 1) - E_0(N)$  is the energy of a charged excitation. In the absence of disorder, it follows from translational invariance[10, 11] that  $A(\epsilon)$  is independent of the single-particle angular momentum label  $m$  and depends only on the filling factor  $\nu = N/N_{LL}$ . ( $N_{LL}$  is the number of states per Landau level in a finite area system.) When energies are measured from the chemical potential, the differential conductivity[9] at bias voltage  $V$  between a LLL 2DEG and a counter-electrode that is separated by a thin tunnel barrier and has a smooth density-of states is equal to  $A(eV)$  up to a constant factor. When an electron is suddenly added to or removed from the ground state, correlations are disturbed and the resulting state is not an eigenstate of the many-electron Hamiltonian. Tunneling spectroscopy experiments measure the energy probability distribution in the sudden state. Note that  $A_+(\epsilon)$  is non-zero only for  $\epsilon > \mu_N = E_0(N+1) - E_0(N)$ , whereas  $A_-(\epsilon)$  is non-zero only for  $\epsilon < \mu(N-1) = E_0(N) - E_0(N-1) \leq \mu(N)$ .

We now derive analytic results for  $A(\epsilon)$  that are valid when the LLL is nearly filled or nearly empty by exploiting particle-hole symmetry[12] and the very simple properties of  $N = 2$  LLL eigenstates. In the absence of disorder, the ground state of a  $N = 1$  LLL system is degenerate; electrons can occupy any angular momentum from  $m = 0$  to  $m = N_{LL} - 1$ . We choose our zero of energy so that the kinetic energy in the LLL is 0. The  $N = 1$  ground state energy is then  $-\Delta_z/2$  where  $\Delta_z$  is the Zeeman splitting between majority and minority spin energies. The electron removal part of the spectral function for  $N = 1$  is a delta-function at this energy with

weight  $N_{LL}^{-1}$ . Two-particle electron states are either the product of a triplet spin state and an orbital state that changes sign under particle interchange, or the product of a singlet spin state and an orbital state that is invariant under particle interchange. Both types of two-particle orbital states are conveniently obtained by repeated application of center-of-mass (COM) and relative angular momentum raising operators[3]:

$$\begin{aligned} b_R^\dagger &= \frac{b_1^\dagger + b_2^\dagger}{\sqrt{2}} \\ b_r^\dagger &= \frac{b_1^\dagger - b_2^\dagger}{\sqrt{2}} \end{aligned} \quad (3)$$

where  $b_i^\dagger$  is the LLL angular momentum raising operator for particle  $i$ ,  $b_R^\dagger$  raises the center-of-mass angular momentum  $M$  and  $b_r^\dagger$  raises the relative angular momentum  $k$ . The  $N = 2$  orbital states labeled by  $M$  and  $k$  are

$$|M, k\rangle = \frac{(b_R^\dagger)^M (b_r^\dagger)^k}{\sqrt{M!k!}} |0, 0\rangle. \quad (4)$$

Here  $|0, 0\rangle$  is the two-particle state in which both individual angular momenta, the COM angular momentum, and the relative angular momentum, are all equal to 0. Since the interaction Hamiltonian acts only on the relative degree-of-freedom and is diagonal in relative angular momenta when the 2DEG is isotropic, these states are two-particle Hamiltonian eigenstates with eigenenergy  $V_k - \Delta_z S_z$ . Here  $S_z$  is the component of total spin along the field direction and  $V_k$ , the expectation value of the pair interaction in relative-angular-momentum state  $k$ , is the  $k$ 'th Haldane pseudopotential. Correlation physics in the quantum Hall regime depends almost entirely on the relative values of the first few Haldane pseudopotentials.

We detail our electron addition spectral function calculation only for the case in which the angular momentum of the added electron  $m_2 = 0$ ; it is easy to verify that identical results are obtained for any value of  $m_2$  as required by translational invariance. Our spectral-function calculation proceeds by averaging over all possible values  $m_1$  of the  $N = 1$  state angular momentum. We consider first the case in which the spin of the added electron is parallel to the  $N = 1$  ground state spin. The anti-symmetrized two-particle state created upon electron addition,

$$|\Psi_{m_2, m_1}\rangle = \frac{(b_1^\dagger)^{m_1} (b_2^\dagger)^{m_2} - (b_1^\dagger)^{m_2} (b_2^\dagger)^{m_1}}{\sqrt{2m_1!m_2!}} |0, 0\rangle, \quad (5)$$

is not an eigenstate of the Hamiltonian. In order to evaluate the spectral function we need to expand this state in terms of the two-particle eigenstates which have definite relative angular momentum  $k$ . This is easily accomplished using the relationship between single-particle and COM-relative angular momentum raising operators (Eq. (3)). After a bit of algebra we find that the probability of obtaining a state with odd relative angular momentum  $k$  upon adding an electron with  $m_2 = 0$  to a

single electron state with angular momentum  $m_1$  is

$$P_k(m_1) = \frac{2}{k!} \frac{m_1!}{2^{m_1}(m_1 - k)!}. \quad (6)$$

(Even relative angular momenta do not appear in the eigenstate expansion of the parallel spin sudden state.) For example, adding a  $m_2 = 0$  electron to a  $N = 1$  state with  $m_1 = 3$  yields a state with relative angular momentum 1 with probability  $P_1(3) = 3/4$  and a state with relative angular momentum 3 with probability  $P_3(3) = 1/4$ . Noting that  $\sum_{m_1} P_k(m_1) = 4$  for all  $k$ , we obtain for the following result for the parallel spin contribution to the  $N = 1$  electron addition spectral function:

$$A_+^{\parallel}(\epsilon) = \frac{4}{N_{LL}} \sum_{k \in \text{odd}} \delta(\epsilon - V_k + \Delta_z/2), \quad (7)$$

where the maximum value of  $k$  is  $N_{LL}/2$ .

When the added electron and the  $N = 1$  ground state electron have opposite spin, the symmetrized two-particle states have a spin-factor that is the  $S_z = 0$  member the two-particle singlet for even  $k$  and the  $S_z = 0$  member of the two-particle triplet for odd  $k$ . The normalization factor of these spin-states leads to a result for  $P_k(m_1)$  that is smaller than the parallel spin result by a factor of two. The end result is that for opposite spin addition and  $N = 1$

$$A_+^{\text{opp}} = \frac{2}{N_{LL}} \sum_k \delta(\epsilon - V_k - \Delta_z/2). \quad (8)$$

The TDOS consists of peaks located, apart from small Zeeman energy contributions, at both even and odd  $k$  Haldane pseudopotential energies.

For values of  $\nu$  close to zero, the typical distance between correlated ground state electrons is large. We can therefore view a system with area  $A$  as consisting of a  $N$  subsystems with area  $A/N$  within which interactions with other electrons can be neglected. The  $N = 1$  results for spectral functions contributions near energy  $V_k$  therefore apply at small but finite  $\nu$  if we replace  $N_{LL}^{-1}$  in Eqs. (7) and (8) by  $(N_{LL}/N)^{-1} = \nu$ , provided that  $k$  is small compared to  $N_{LL}/2N = 1/2\nu$ . Added electrons will have a significant probability of forming high-energy small- $k$  relative angular momentum states only if they are added at a position close to that of a ground state electron, and this becomes more likely as  $\nu$  increases. It follows that a peak in the spectral function with weight proportional to  $\nu$  is expected near  $\epsilon = V_k$  over a range of  $\nu$  that decreases with  $k$ .

In the tunneling data of Dial and Ashoori[5] only the  $k = 0$  and  $k = 1$  peaks are apparent. Higher  $k$  peaks that would be expected to be visible over narrower ranges of  $\nu$  are evidently obscured by disorder, which is of course present even in these very high quality samples. We refer to these peaks in the TDOS as *Haldane sashes*. From Ref.[5] we can read off experimental values for the  $k = 0$  and  $k = 1$  Haldane pseudopotentials:  $V_0 \approx 9meV$ , and

$V_1 \approx 6\text{meV}$ . The  $V_1$  peak feature appears to be approximately three times stronger than the higher energy  $V_0$  peak feature as predicted by this theoretical analysis.

The Haldane sashes should broaden as  $\nu$  increases. They are nevertheless evident over a fairly broad range of  $\nu$  and provide valuable qualitative insight into LLL electronic correlations. The  $k = 1$  sash feature, for example, is evident up to  $\nu = 1/3$  where its lower edge approaches the Fermi level. This behavior is expected since the possibility of making charged excitations that avoid relative-angular-momentum  $k = 1$  is absent[3, 13] for  $\nu > 1/3$ . Similarly the  $k = 0$  feature, associated with adding an opposite spin electron at the same position as an existing ground state electron, is evident for all  $\nu \leq 1$ . At  $\nu = 1$  its lower edge approaches the Fermi energy. The  $k = 1$  sash is not visible over the range  $1/3 < \nu < 2/3$  where composite-fermion physics reigns, demonstrating that the relationship between spectra and spatial correlations is more subtle in the composite fermion regime. Although composite fermion physics is not revealed by the  $T = 0$  TDOS, we anticipate that it will emerge in its  $T$ -dependence as explained below.

Provided that the ground state is maximally spin-polarized at all filling factors[14], particle-hole symmetry[11, 12] implies that the parallel spin contribution to  $A_+(\epsilon)$  at filling factor  $\nu$  equals  $A_-(-\epsilon)$  at filling factor  $1 - \nu$ . It follows that hole-pair Haldane sashes are present in the electron removal part of the spectral function near  $\nu = 1$ . Hole-pair Haldane sashes are also evident in Ref.[5].

*TDOS and Correlation Energies*—The progress reported in Ref.[5] motivates a reexamination of the relationship between the TDOS and correlation energies discussed previously[11] by Haussmann *et al.*. This relationship is simplest when only one spin-component is involved and, for this reason, we now focus on the electron-removal TDOS for  $\nu < 1$ . Recognizing that energies are in general known only relative to the chemical potential, we rewrite the energy expression derived by Haussmann *et al.* in the form

$$\int_{-\infty}^{\infty} d\xi \xi A_-(\xi) n_F(\xi) \equiv S_-(\nu) = 2\epsilon_0(\nu) - \mu(\nu), \quad (9)$$

where  $\xi$  is energy measured from the chemical potential,  $n_F(\xi) = \theta(-\xi)$  is the  $T = 0$  Fermi factor,  $\epsilon_0(\nu)$  is the ground state energy per LLL state relative to the LLL single-particle energy and  $\mu = \epsilon'_0(\nu)$  is the chemical potential. Note that in evaluating  $A_-(\xi)$  from experimental tunneling data, uncertainty related to tunneling matrix-element factors can be mitigated by applying the sum rule

$$\int_{-\infty}^0 d\xi A_-(\xi) n_F(\xi) = \nu. \quad (10)$$

Eq.( 9) can be rewritten in the form,

$$\frac{d}{d\nu} \frac{\epsilon_0(\nu)}{\nu^2} = -\frac{S_-(\nu)}{\nu^3}, \quad (11)$$

and integrated over filling factor to extract the filling factor dependence of the ground state energy and chemical potential directly from the tunneling data. For very small  $\nu$  we expect the ground state energy to be given accurately by the classical triangular lattice value[15]

$$\epsilon_0(\nu) \rightarrow \epsilon_{WC}(\nu) = -0.782133 \frac{e^2}{\epsilon \ell} \nu^{3/2}. \quad (12)$$

where  $\ell$  is the magnetic length. It follows that in the same limit  $S_-(\nu) \propto \nu^{3/2}$  and that integrals of  $S_-(\nu)/\nu^3$  do not converge when the lower limit extends to  $\nu = 0$ . (The  $\nu^{3/2}$  small  $\nu$  behavior of  $S_-(\nu)$  is evident in Ref.[5].) Ground state energies must therefore be determined either by connecting to the classical triangular lattice value at a small filling factor  $\nu^*$  using,

$$\epsilon_0(\nu) = \nu^2 \left[ \frac{\epsilon_{WC}(\nu^*)}{(\nu^*)^2} - \int_{\nu^*}^{\nu} d\nu' \frac{S_-(\nu')}{\nu'^3} \right], \quad (13)$$

or by relating the correlated fractional filling factor ground state energy to the full Landau level ground state energy using

$$\epsilon_0(\nu) = \nu^2 \left[ \epsilon_0(\nu = 1) + \int_{\nu}^1 d\nu' \frac{S_-(\nu')}{\nu'^3} \right]. \quad (14)$$

Once  $\epsilon_0(\nu)$  is known,  $\mu(\nu)$  can be obtained using Eq.( 9). *Discussion*—Correlation physics in a Landau level is known to be highly sensitive to Haldane pseudopotential  $V_k$  values; the qualitative differences between  $n = 0$  composite-fermion,  $n = 1$  non-abelian quasiparticle, and  $n \geq 2$  density-wave physics are associated with relatively modest changes in  $V_k$  ratios. We have shown that tunneling spectroscopy can be used to measure the most important  $V_k$  values. Since the numerical values of these parameters can be difficult to estimate accurately on the basis of theoretical considerations because of uncertainties related to quantum well width and quantum fluctuations involving higher Landau levels and higher subbands, the ability to measure their values using tunneling spectroscopy is a valuable advance. The numerical values measured by Dial and Ashoori,  $V_0 \approx 9\text{meV}$  and  $V_1 \approx 6\text{meV}$ , are in the expected range.

One important experimental finding of Dial and Ashoori is that Haldane sashes survive only up to temperatures that are quite small compared to the energies at which they appear. This behavior is expected since, as we have explained, the sharpness of the Haldane sashes is dependent on correlations which keep the electrons well separated. The small filling factor Wigner crystal state, for example, melts[16] when  $k_B T \gtrsim 0.005(e^2/\epsilon \ell)\nu^{1/2}$ . Sash features in the TDOS reflect strong correlations which develop only as the crystallization temperature is approached.

The temperature dependence of the TDOS can also shed light[18] on thermodynamic properties that are normally not accessible in 2DES's and could lead to valuable new insights into LLL correlation physics. Since Eq.( 9)

remains valid[9] at finite  $T$ , tunneling data can be used to determine  $\epsilon_0(\nu)$  and  $\mu(\nu)$  as a function of  $T$ . The temperature dependence of energy yields the heat capacity,

$$C = N_{LL} \frac{\partial \epsilon_0}{\partial T} = T \frac{\partial S}{\partial T}, \quad (15)$$

and therefore the temperature-dependence of the entropy. Unlike the TDOS, the heat capacity and the entropy are determined by the excitation spectrum of the system alone, and are not influenced by the overlap[17] between charged excitations and bare electron excitations. According to composite fermion theory, the heat capacity at Landau level filling factor  $\nu = 1/2$  is given by

$$\frac{C}{N_{LL}} = \frac{\pi^2 k_B^2 T}{3} \frac{m_{cf}^* \ell^2}{\hbar^2} \quad (16)$$

where  $m_{cf}^*$  is the composite fermion effective mass. Verification of Eq.( 16) could directly establish the composite fermion ansatz. The property that the heat capacity depends on  $m_{cf}^*$  alone, independent of complex quasiparticle normalization factors, is analogous to the corresponding Fermi liquid property.

The temperature-dependence of the chemical potential could also be very revealing, especially for incompressible

states like the one thought to be responsible for the quantum Hall effect at filling factor  $\nu = 5/2$  ( $\nu = 1/2$  in the  $n = 1$  Landau level), that have non-Abelian quasiparticles. The theoretically expected quasiparticle statistics imply[19, 20] finite entropy,

$$S = \frac{k_B |2N - N_{LL}| \ln 2}{2}, \quad (17)$$

above exponentially small temperatures at fractional filling factors near  $\nu = 1/2$  within a  $n = 1$  level. The dependence of entropy on particle number, which changes sign when  $\nu$  crosses  $1/2$ , can be obtained directly from chemical potential data using the thermodynamic identity

$$\frac{\partial S}{\partial N} = \frac{\partial \mu}{\partial T}. \quad (18)$$

*Acknowledgements*— AHM thanks Oliver Dial and Ray Ashoori for a stimulating conversation during EP2DS18. This work was supported by the Welch Foundation and by the National Science Foundation under grant DMR-0606489.

- 
- [1] For a review see Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma, *Rev. Mod. Phys.* **80**, 1083 (2008).
  - [2] For a review see Eduardo Fradkin, Steven A. Kivelson, Michael J. Lawler, James P. Eisenstein, and Andrew P. Mackenzie, arXiv:0910.4166 (to appear in *Annual Reviews in Condensed Matter Physics*).
  - [3] See for example A.H. MacDonald in *Les Houches Summer School Session* **61**, 659 (1995).
  - [4] J.K. Jain, *Phys. Rev. Lett.* **63**, 199 (1989); B.I. Halperin, P.A. Lee, and N. Read, *Phys. Rev. B* **47**, 7312 (1993).
  - [5] O.E. Dial, R.C. Ashoori, L.N. Pfeiffer, and K.W. West, *Nature* **464**, 566 (2010).
  - [6] J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, *Phys. Rev. Lett.* **69**, 3804 (1992); *Surf. Sci.* **305**, 393 (1994); *Phys. Rev. Lett.* **74**, 1419 (1995); *Solid State Commun.* **149**, 1867 (2009).
  - [7] R.C. Ashoori, J.A. Lebens, N.P. Bigelow, and R.H. Silsbee, *Phys. Rev. Lett.* **64**, 681 (1990); *Phys. Rev. B* **48**, 4616 (1993).
  - [8] F.D.M. Haldane, *Phys. Rev. Lett.* **51**, 605 (1983).
  - [9] See for example G.D. Mahan, *Many-Particle Physics*, (Plenum, New York, 1990).
  - [10] We couch our discussion in terms of symmetric gauge LLL states in which single-particle states have non-negative integer angular momentum labels.
  - [11] Rudolf Haussmann, Hiroyuki Mori, and A.H. MacDonald, *Phys. Rev. Lett.* **76**, 980 (1996).
  - [12] A.H. MacDonald and E.H. Rezayi, *Phys. Rev. B* **42**, 3224 (1990).
  - [13] R.B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983).
  - [14] We assume maximal spin polarization throughout this article. In GaAs 2DEGs this condition is typically satisfied at fields above  $\sim 8$  Tesla. See for example R.R. Du, A.S. Yeh, H.L. Stormer *et al.*, *Phys. Rev. Lett.* **75**, 3926 (1995); M.J. Manfra, E.H. Aifer, B.B. Goldberg, *et al.*, *Phys. Rev. B* **54**, 17327 (1996); O. Stern *et al.*, *Phys. Rev. B* **70**, 075318 (2004); S. Melinte, N. Freytag, M. Horvatic *et al.*, *Phys. Rev. Lett.* **84**, 354 (2000).
  - [15] L. Bonsall and A.A. Maradudin, *Phys. Rev. B* **15**, 1959 (1977).
  - [16] R.H. Morf, *Phys. Rev. Lett.* **43**, 931 (1979).
  - [17] These overlaps tend to be small in strongly correlated electron systems, including quantum Hall systems, and tend to suppress the TDOS at low energies. See Y. Hatsugai, P.-A. Bares, and X. G. Wen, *Phys. Rev. Lett.* **71**, 424 (1993); S. He, P. M. Platzman, and B. I. Halperin, *Phys. Rev. Lett.* **71**, 777 (1993); P. Johansson and J. M. Kinaret, *Phys. Rev. Lett.* **71**, 1435 (1993); A. L. Efros and F. G. Pikus, *Phys. Rev. B* **48**, 14694 (1993); C. M. Varma, A. I. Larkin, and E. Abrahams, *Phys. Rev. B* **49**, 13999 (1994); I. L. Aleiner, H. U. Baranger, and L. I. Glazman, *Phys. Rev. Lett.* **74**, 3435 (1995).
  - [18] The differential TDOS at finite temperature is proportional to  $A(\xi)$  averaged over energy with a  $\partial n_F(\xi)/\partial \xi$  weighting factor. It is in principle possible if necessary to correct for the thermal broadening and extract the underlying temperature-dependent spectral function.
  - [19] N.R. Cooper and Ady Stern, *Phys. Rev. Lett.* **102**, 176807 (2009).

- [20] K. Yang and B.I. Halperin, Phys. Rev. B **79**, 115317 (2009).